# Problem Corner

Readers are welcome to submit solutions by email to `princetonmathjournal@gmail.com`. Successful solvers will be commended in the next issue.

1. Let $n \in \mathbb{N}$ be a fixed positive integer, and consider a finite field $\mathbb{F} = \mathbb{F}_q$ with $q$ elements. Define $f_n(q)$ to be the number of ordered pairs of invertible commuting $n \times n$ matrices $(A, B)$.
   (That is, $A, B \in \mathrm{GL}_n(\mathbb{F}_q)$ and $AB = BA$).

   Prove that $f_n(q)$ is a monic polynomial in $q$ of degree $n^2 + n$.

   *Proposed by Iurie Boreico, Stanford University, USA*

2. Consider an $N \times N$ grid of unit squares which are initially all colored white. By a "line of squares" in the grid we mean either a row or column of squares.

   Two players $A$ and $B$ play the following game on the grid. The players alternate turns with $A$ going first. On the first move, $A$ chooses any line $l$. On the second move, $B$ must select a square on the line $l$ and color it black. Then $B$ must choose a line which does not pass through the black square. On subsequent moves, a player must first select a square on the line his opponent chose the previous turn and color it black, then choose a new line not containing any black square.

   The loser is the first player who cannot choose a line missing the black squares. Is there a winning strategy for some player? If yes, find it.

   *Proposed by Cosmin Pohoata, Columbia University, USA*

3. Find all integer solutions to the equation $x^3 + y^3 + z^3 = 33$.

   *Proposed by Andy Loo, Princeton University, USA,*
   *who heard of it from Professor John H. Conway, Princeton University, USA*